Optimum Optical Systems

Future Accelerators

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PS SEMINAR

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I - Optimum Optical Systems

Principles of Symbolic Optics with *BeamOptics*

Optimum Optical Solution, Examples

II - Design for Future Accelerators

Neutrino Factory

Design Options

BeamOptics

"*BeamOptics*, A Program for Analytical Beam Optics"
B. Autin (editor), C. Carli, T. D'Amico, O. Gröbner, M. Martini, E. Wildner
Yellow Report CERN 98-06

A - Reminder of some Principles of Transverse Optics

B - Basic Optical Elements

C - Constraints, Boundary Conditions

D - General Method with Optical Modules, Examples

E - Optimum Optical Modules, Examples

Principles of Symbolic Optics 1

- In a curvilinear system, the equations of motion in the transverse plane are second order Hill's equations:

$$x'' + k(s)x = f(s)$$

where s is the curvilinear abscissa and $'$ the derivative with respect to s.

- Coordinates mapping through an optical element is linear and characterized by a transfer matrix.

- Pure quadrupole of length l and focusing strength k:

$$M = \begin{pmatrix} \cos\sqrt{k}l & \frac{\sin\sqrt{k}l}{\sqrt{k}} \\ -\sqrt{k}\sin\sqrt{k}l & \cos\sqrt{k}l \end{pmatrix} \quad , \quad \begin{cases} k > 0 \text{ focusing} \\ k < 0 \text{ defocusing} \end{cases}$$

Principles of Symbolic Optics 2

- Betatron oscillations are described by the Courant-Snyder theory using the phase-amplitude formalism.
- The solution to the homogeneous linear second-order equation is a linear combination of two functions:

$$x(s) = \sqrt{\epsilon\beta(s)}(x_0 \cos\mu(s) + x_0' \sin\mu(s))$$

where $\epsilon = x_0^2 + x_0'^2$ is the Courant-Snyder invariant.

- Characteristic functions:

$$\beta(s) \quad , \quad \frac{d\mu(s)}{ds} = \frac{1}{\beta(s)} \quad , \quad \alpha(s) = -\frac{1}{2}\frac{d\beta(s)}{ds} \quad , \quad \gamma(s) = \frac{1 + \alpha(s)^2}{\beta(s)}$$

Principles of Symbolic Optics 3

- The envelope of betatron oscillations is described by the functions β and α.
- Transverse information gathered in the σ matrix:

$$\sigma = \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

- Tracing β- and α-functions through an optical element i of transfer matrix M_i results from the recursion of the σ matrix:

$$\sigma_i = M_i \sigma_{i-1} M_i^t$$

- Courant-Snyder invariant ϵ is:

$$X^t \sigma^{-1} X \quad \text{with} \quad X = \begin{pmatrix} x \\ x' \end{pmatrix}$$

Basic Optical Elements

- In paraxial optics, three types of objects are considered:
  Drift spaces, Bending magnets, Quadrupoles

- The objects are defined by polymorphic functions:
  - ▷ *Q[f]* is a thin quadrupole of focal length f

    Q[k,l] is a quadrupole of length l and of strength k
  - ▷ *SS[l]* is a drift space of lengh l
  - ▷ *Bend[ϕ]* is a thin bending magnet of deflection angle ϕ

    Bend[l,ϕ] is a bending magnet of orbit length l and deflection ϕ

Constraints

For each type of module, there are initial or cyclic boundary conditions:

→ Betatron Matching with a doublet:

- Crossover conditions: $\beta_h = \beta_v \ , \ \alpha_h = -\alpha_v$
- Waist conditions: $\beta_h = \beta_v \ , \ \alpha_h = \alpha_v = 0$

→ Betatron Matching with a triplet has no constraints

→ Periodic structures require periodic betatron functions and dispersion

General Method with Optical Modules

The method consists in breaking up a complex structure into smaller, independent modules:

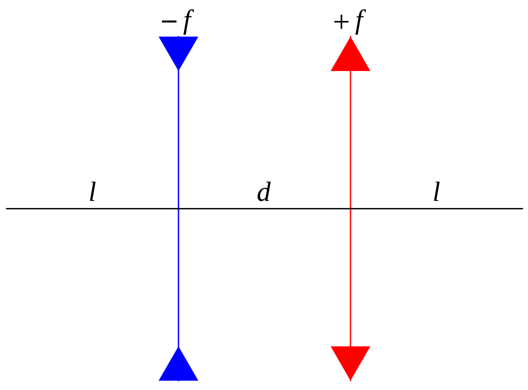
1) In the thin lens model, the problem is deterministic and exact solutions are found.
2) A module made of real elements is automatically built in the neighborhood of the thin lens solution.
3) The final structure is an assembly of modules.

Different types of modules:

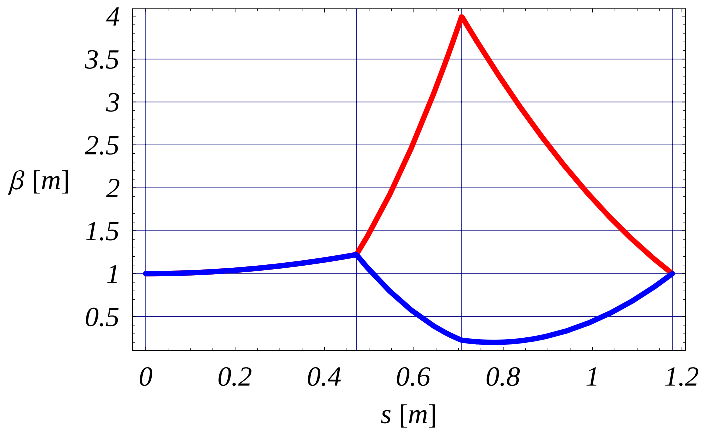
▷ Betatron Matching Modules (Doublet, Triplet, $\lambda/4$ Transformer)
▷ Periodic Structures (FODO Cell, Isochronous Period, Collins Insertion)
▷ Telescopic Systems (Telescope, Inversor)
▷ Orbit Matching Modules (Dispersion Suppressor)

Betatron Matching with *BeamOptics*

MatchingDoublet[Sigma[β_1, α_1], Sigma[β_2, α_2]]



$\beta_1 = 1$, $\alpha_1 = 0$ and $\beta_2 = 1$, $\alpha_2 = 2$



$$f = -\frac{\alpha_1 + \alpha_2}{\gamma_1 + \gamma_2}$$

$$l = \frac{\sqrt{\beta_1\gamma_2 + \beta_2\gamma_1 + 2(1 - \alpha_1\alpha_2)}}{\gamma_1 + \gamma_2}$$

$$f = \sqrt{dl}$$

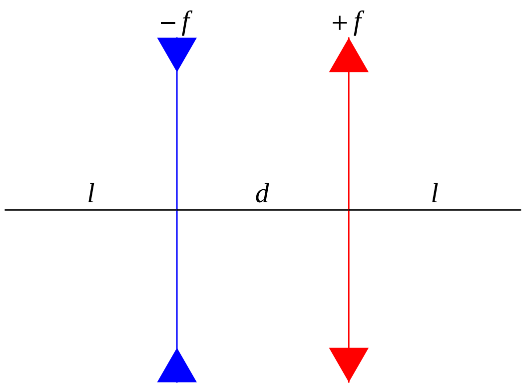
Horizontal and vertical β-functions in the thin lens model

Symbolic Calculations with *BeamOptics*

Algebraic expressions of α- and β-functions are extracted

Number of variables = Number of constraints

scaling parameters, simplification




```
In[27]:= ch = Channel[SS[l], Q[-f], SS[d], Q[f], SS[l]];
         SigmaAt[ch, Sigma[1, 0]]
```

Out[28]=

$$\mathrm{Sigma}\Big[\frac{1}{f^4}\big(d^2 f^2 + 2 d f^3 + f^4 + d^2 f^4 - 2 d^2 f l - 2 d f^2 l + 4 d f^4 l + d^2 l^2 - 2 d^2 f^2 l^2 + 4 f^4 l^2 - 4 d f^2 l^3 + d^2 l^4\big),\ \frac{d^2 f + d f^2 + d^2 f^3 - d f^4 - d^2 l + d^2 f^2 l + 2 d f^3 l - 2 f^4 l - d^2 f l^2 + 3 d f^2 l^2 - d^2 l^3}{f^4}\Big]$$

```
In[29]:= SigmaAt[ch, Sigma[a, b]]
```

Out[29]=

$$\mathrm{Sigma}\Big[\frac{1}{a f^4}\big(a^2 d^2 f^2 + 2 a^2 d f^3 - 2 a b d^2 f^3 + a^2 f^4 - 2 a b d f^4 + d^2 f^4 + b^2 d^2 f^4 - 2 a^2 d^2 f l - 2 a^2 d f^2 l + 2 a b d^2 f^2 l - 4 a b d f^3 l - 4 a b f^4 l + 4 d f^4 l + 4 b^2 d f^4 l + a^2 d^2 l^2 + 2 a b d^2 f l^2 + 6 a b d f^2 l^2 - 2 d^2 f^2 l^2 - 2 b^2 d^2 f^2 l^2 + 4 f^4 l^2 + 4 b^2 f^4 l^2 - 2 a b d^2 l^3 - 4 d f^2 l^3 - 4 b^2 d f^2 l^3 + d^2 l^4 + b^2 d^2 l^4\big),\ \frac{1}{a f^4}\big(a^2 d^2 f + a^2 d f^2 - 2 a b d^2 f^2 + d^2 f^3 + b^2 d^2 f^3 + a b f^4 - d f^4 - b^2 d f^4 - a^2 d^2 l - 4 a b d f^2 l + d^2 f^2 l + b^2 d^2 f^2 l + 2 d f^3 l + 2 b^2 d f^3 l - 2 f^4 l - 2 b^2 f^4 l + 2 a b d^2 l^2 - d^2 f l^2 - b^2 d^2 f l^2 + 3 d f^2 l^2 + 3 b^2 d f^2 l^2 - d^2 l^3 - b^2 d^2 l^3\big)\Big]$$

Periodic Structures

- For a period of length L:

$$\beta(s+L) = \beta(s) \quad , \quad \alpha(s+L) = \alpha(s) \quad , \quad D(s+L) = D(s)$$

- If Q is the tune, the transfer matrix becomes:

$$M_0 = I \cos 2\pi Q + S_0 \sin 2\pi Q \;\; \text{with} \;\; S_0 = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix}$$

- By identification,

$$\beta = \frac{m_{12}}{\sin 2\pi Q} \quad , \quad \alpha = \frac{m_{11} - m_{22}}{\sin 2\pi Q} \quad , \quad D = \frac{m_{13}(1 - m_{22}) + m_{12} m_{23}}{2(1 - \cos 2\pi Q)}$$

Isochronous Period with *BeamOptics* (1)

- Structure tuned near an integer by removing some magnets in a regular arrangement
- Negative dispersion creates negative momentum compaction

$$\alpha_p = \frac{\Delta L/L}{\Delta p/p} = \frac{1}{L}\int_0^L \frac{D}{\rho} ds$$

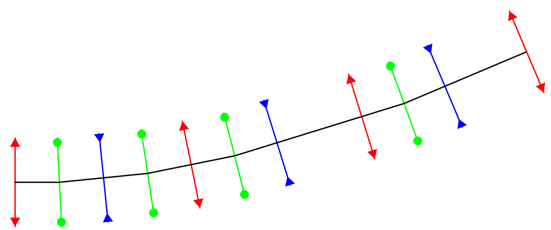
where L is the path length, p the momentum, ρ the radius of curvature

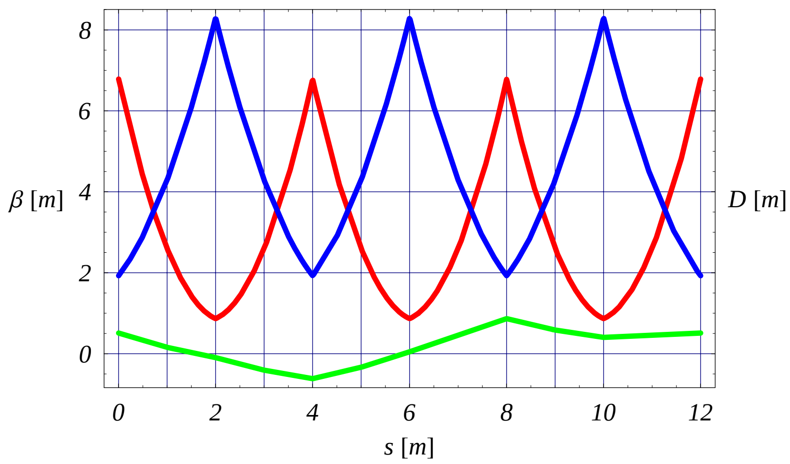
- Over the period, the contributions of the velocity and of the orbit length to the revolution frequency variation cancel each other
- Necessary to preserve the bunch length (e.g. CLIC combiner ring)

ISOCHRONOUS PERIOD WITH *BeamOptics* (2)

IsoPeriod[n, MuV, Deflection, CellLength, MissingMagnet]

- Focusing and dispersion are decoupled
- β-functions are the same as in a FODO cell
- Dispersion is influenced only by the distribution of the bending magnets





Horizontal and vertical β-functions and dispersion in thin lens model for n=3

Optimum Optical Modules

- Build an analytic channel made of real, finite length lenses
- Derive an analytical, quadratic, χ^2-type function including all the parameters of the structure
- Choose the variables of the function
- The initial values for minimization are given algebraically by the thin lens model
- Numerical minimization of the function (transcendental equations)
- Result: automatic elaboration of optical modules made of finite length elements with numerical and graphical output

General Conditions

- For real systems, canonical conditions may be slightly violated
- Ratio of quadrupole length to quadrupole spacing (between center)

$$\frac{l_q}{d} < 1$$

$$
\begin{aligned}
&u[k\_,\ L\_,\ x\_,\ y\_,\ \alpha\_,\ \beta\_,\ a\_,\ b\_] = \\
&\Bigl(\frac{k^2 x^2 y^3}{2\beta} + \frac{k^2 x^2 y^3 \alpha^2}{2\beta} + \frac{1}{2} k^2 x^2 y \beta + \text{Cosh}[2\sqrt{k}\,L] \\
&\quad \Bigl(-\frac{x}{2\beta} - \frac{y}{\beta} - \frac{k x^2 y}{2\beta} - \frac{3 k x y^2}{2\beta} - \frac{k y^3}{\beta} - \frac{k^2 x^2 y^3}{2\beta} - \frac{x\alpha^2}{2\beta} - \frac{y\alpha^2}{\beta} - \frac{k x^2 y \alpha^2}{2\beta} - \\
&\qquad \frac{3 k x y^2 \alpha^2}{2\beta} - \frac{k y^3 \alpha^2}{\beta} - \frac{k^2 x^2 y^3 \alpha^2}{2\beta} - \frac{k x \beta}{2} - k y \beta - \frac{1}{2} k^2 x^2 y \beta + \\
&\qquad \Bigl(-\frac{1}{2\sqrt{k}\,\beta} + \frac{\sqrt{k}\,x^2}{4\beta} + \frac{2\sqrt{k}\,x y}{\beta} + \frac{3\sqrt{k}\,y^2}{2\beta} + \frac{3 k^{3/2} x^2 y^2}{4\beta} + \frac{k^{3/2} x y^3}{\beta} - \\
&\qquad\quad \frac{\alpha^2}{2\sqrt{k}\,\beta} + \frac{\sqrt{k}\,x^2\alpha^2}{4\beta} + \frac{2\sqrt{k}\,x y \alpha^2}{\beta} + \frac{3\sqrt{k}\,y^2\alpha^2}{2\beta} + \frac{3 k^{3/2} x^2 y^2 \alpha^2}{4\beta} + \\
&\qquad\quad \frac{k^{3/2} x y^3 \alpha^2}{\beta} + \frac{\sqrt{k}\,\beta}{2} + \frac{1}{4} k^{3/2} x^2 \beta + k^{3/2} x y \beta\Bigr) \text{Sin}[2\sqrt{k}\,L]\Bigr) + \\
&\quad \Bigl(-\frac{1}{2\sqrt{k}\,\beta} - \frac{\sqrt{k}\,x^2}{4\beta} - \frac{\sqrt{k}\,x y}{\beta} - \frac{3\sqrt{k}\,y^2}{2\beta} - \frac{3 k^{3/2} x^2 y^2}{4\beta} - \frac{k^{3/2} x y^3}{\beta} - \\
&\qquad \frac{\alpha^2}{2\sqrt{k}\,\beta} - \frac{\sqrt{k}\,x^2\alpha^2}{4\beta} - \frac{\sqrt{k}\,x y \alpha^2}{\beta} - \frac{3\sqrt{k}\,y^2\alpha^2}{2\beta} - \frac{3 k^{3/2} x^2 y^2 \alpha^2}{4\beta} - \\
&\qquad \frac{k^{3/2} x y^3 \alpha^2}{\beta} - \frac{\sqrt{k}\,\beta}{2} - \frac{1}{4} k^{3/2} x^2 \beta - k^{3/2} x y \beta\Bigr) \text{Sinh}[2\sqrt{k}\,L] + \\
&\quad \text{Sin}[2\sqrt{k}\,L]\ \Bigl(-\frac{1}{2\sqrt{k}\,\beta} + \frac{\sqrt{k}\,x^2}{4\beta} + \frac{\sqrt{k}\,x y}{\beta} + \frac{3\sqrt{k}\,y^2}{2\beta} - \frac{3 k^{3/2} x^2 y^2}{4\beta} - \\
&\qquad \frac{k^{3/2} x y^3}{\beta} - \frac{\alpha^2}{2\sqrt{k}\,\beta} + \frac{\sqrt{k}\,x^2\alpha^2}{4\beta} + \frac{\sqrt{k}\,x y \alpha^2}{\beta} + \frac{3\sqrt{k}\,y^2\alpha^2}{2\beta} - \frac{3 k^{3/2} x^2 y^2 \alpha^2}{4\beta} - \\
&\qquad \frac{k^{3/2} x y^3 \alpha^2}{\beta} + \frac{\sqrt{k}\,\beta}{2} - \frac{1}{4} k^{3/2} x^2 \beta - k^{3/2} x y \beta + \Bigl(\frac{k x^2 y}{\beta} + \frac{3 k x y^2}{\beta} + \frac{k y^3}{\beta} + \\
&\qquad\quad \frac{k x^2 y \alpha^2}{\beta} + \frac{3 k x y^2 \alpha^2}{\beta} + \frac{k y^3 \alpha^2}{\beta} + k x \beta + k y \beta\Bigr) \text{Sinh}[2\sqrt{k}\,L]\Bigr) + \\
&\quad \text{Cos}[2\sqrt{k}\,L]\ \Bigl(-\frac{x}{2\beta} - \frac{y}{\beta} + \frac{k x^2 y}{2\beta} + \frac{3 k x y^2}{2\beta} + \frac{k y^3}{\beta} - \frac{k^2 x^2 y^3}{2\beta} - \\
&\qquad \frac{x\alpha^2}{2\beta} - \frac{y\alpha^2}{\beta} + \frac{k x^2 y \alpha^2}{2\beta} + \frac{3 k x y^2 \alpha^2}{2\beta} + \frac{k y^3 \alpha^2}{\beta} - \frac{k^2 x^2 y^3 \alpha^2}{2\beta} + \\
&\qquad \frac{k x \beta}{2} + k y \beta - \frac{1}{2} k^2 x^2 y \beta + \Bigl(-\frac{x}{\beta} - \frac{2 y}{\beta} + \frac{k^2 x^2 y^3}{2\beta} - \frac{x\alpha^2}{\beta} - \\
&\qquad\quad \frac{2 y \alpha^2}{\beta} + \frac{k^2 x^2 y^3 \alpha^2}{2\beta} + \frac{1}{2} k^2 x^2 y \beta\Bigr) \text{Cosh}[2\sqrt{k}\,L] + \\
&\qquad \Bigl(-\frac{1}{2\sqrt{k}\,\beta} - \frac{\sqrt{k}\,x^2}{4\beta} - \frac{2\sqrt{k}\,x y}{\beta} - \frac{3\sqrt{k}\,y^2}{2\beta} + \frac{3 k^{3/2} x^2 y^2}{4\beta} + \frac{k^{3/2} x y^3}{\beta} - \\
&\qquad\quad \frac{\alpha^2}{2\sqrt{k}\,\beta} - \frac{\sqrt{k}\,x^2\alpha^2}{4\beta} - \frac{2\sqrt{k}\,x y \alpha^2}{\beta} - \frac{3\sqrt{k}\,y^2\alpha^2}{2\beta} + \frac{3 k^{3/2} x^2 y^2 \alpha^2}{4\beta} + \\
&\qquad\quad \frac{k^{3/2} x y^3 \alpha^2}{\beta} - \frac{\sqrt{k}\,\beta}{2} + \frac{1}{4} k^{3/2} x^2 \beta + k^{3/2} x y \beta\Bigr) \text{Sinh}[2\sqrt{k}\,L]\Bigr)\Bigr)\text{^}2 + \\
&\Bigl(k^2 x^2 y^3 \alpha + \frac{1}{2} k x^2 \beta + \text{Cosh}[2\sqrt{k}\,L]\ \Bigl(-x\alpha - 2 y\alpha - k x^2 y \alpha - 3 k x y^2 \alpha - 2 k y^3 \alpha - \\
&\qquad k^2 x^2 y^3 \alpha + k x y \beta + \Bigl(-\frac{\alpha}{\sqrt{k}} + \frac{1}{2}\sqrt{k}\,x^2\alpha + 4\sqrt{k}\,x y \alpha + 3\sqrt{k}\,y^2\alpha + \frac{3}{2} k^{3/2} x^2 \\
&\qquad\quad y^2\alpha + 2 k^{3/2} x y^3 \alpha - \sqrt{k}\,x\beta - \sqrt{k}\,y\beta + \frac{1}{2} k^{3/2} x^2 y \beta\Bigr) \text{Sin}[2\sqrt{k}\,L]\Bigr) + \\
&\quad \Bigl(-\frac{\alpha}{\sqrt{k}} - \frac{1}{2}\sqrt{k}\,x^2\alpha - 2\sqrt{k}\,x y \alpha - 3\sqrt{k}\,y^2\alpha - \frac{3}{2} k^{3/2} x^2 y^2 \alpha - \\
&\qquad 2 k^{3/2} x y^3 \alpha + \sqrt{k}\,y\beta + \frac{1}{2} k^{3/2} x^2 y \beta\Bigr) \text{Sinh}[2\sqrt{k}\,L] + \text{Sin}[2\sqrt{k}\,L] \\
&\quad \Bigl(-\frac{\alpha}{\sqrt{k}} + \frac{1}{2}\sqrt{k}\,x^2\alpha + 2\sqrt{k}\,x y \alpha + 3\sqrt{k}\,y^2\alpha - \frac{3}{2} k^{3/2} x^2 y^2 \alpha - 2 k^{3/2} x y^3 \alpha + \\
&\qquad \sqrt{k}\,y\beta - \frac{1}{2} k^{3/2} x^2 y \beta + (2 k x^2 y \alpha + 6 k x y^2 \alpha + 2 k y^3 \alpha - \beta)\ \text{Sinh}[2\sqrt{k}\,L]\Bigr) + \\
&\quad \text{Cos}[2\sqrt{k}\,L]\ \Bigl(-x\alpha - 2 y\alpha + k x^2 y \alpha + 3 k x y^2 \alpha + 2 k y^3 \alpha - k^2 x^2 y^3 \alpha + \\
&\qquad k x y \beta + \bigl(-2 x\alpha - 4 y\alpha + k^2 x^2 y^3 \alpha - \frac{1}{2} k x^2 \beta - 2 k x y \beta\bigr) \text{Cosh}[2\sqrt{k}\,L] + \\
&\qquad \Bigl(-\frac{\alpha}{\sqrt{k}} - \frac{1}{2}\sqrt{k}\,x^2\alpha - 4\sqrt{k}\,x y \alpha - 3\sqrt{k}\,y^2\alpha + \frac{3}{2} k^{3/2} x^2 y^2 \alpha + \\
&\qquad\quad 2 k^{3/2} x y^3 \alpha - \sqrt{k}\,x\beta - \sqrt{k}\,y\beta - \frac{1}{2} k^{3/2} x^2 y \beta\Bigr) \text{Sinh}[2\sqrt{k}\,L]\Bigr)\Bigr)\text{^}2;
\end{aligned}
$$

Example with Matching Doublet

Flexibility of the method, Case with multiple solutions

- For a perfect crossover close to thin lens solution:

  $$U(k, l_q, d, l, \alpha, \beta) = (\beta_h - \beta_v)^2 + (\alpha_h + \alpha_v)^2$$

  Purely algebraic, one free parameter: k

- For a crossover sticking to the requested values β_0 and α_0:

  $$U(k, l_q, d, l, \alpha, \beta) = (\beta_h - \beta_0)^2 + (\beta_v - \beta_0)^2 + (\alpha_h - \alpha_0)^2 + (\alpha_v + \alpha_0)^2$$

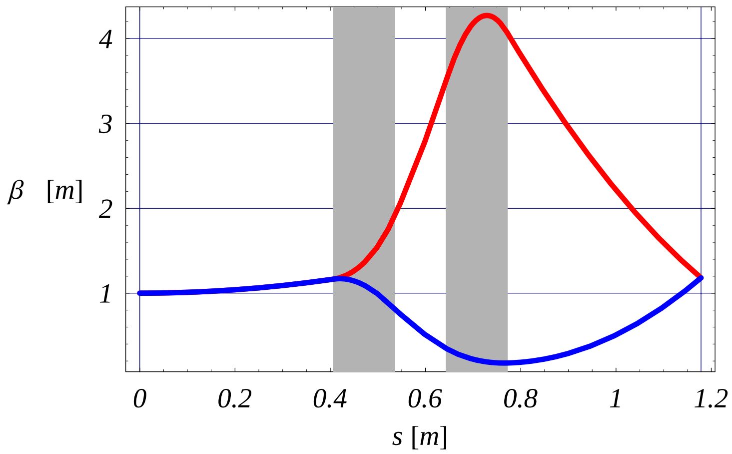
  More conditions, more free parameters needed

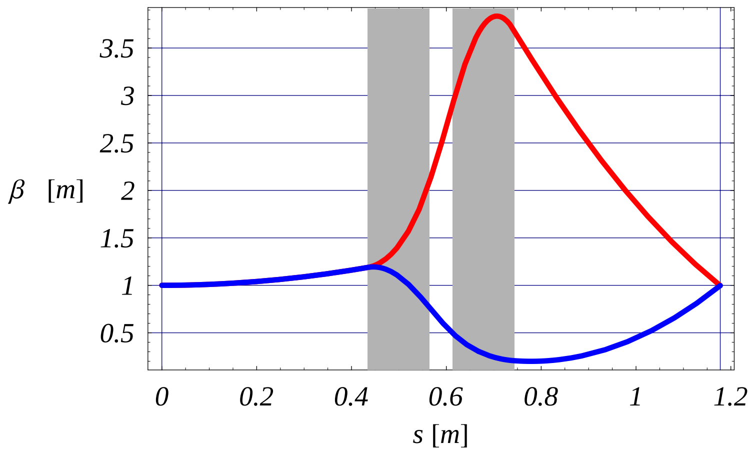
  The greater the number of parameters, the longer the computation time

- Starting point given by $f = -\frac{\alpha_1+\alpha_2}{\gamma_1+\gamma_2}$ and $k = \frac{1}{f * l_q}$

Example with Matching Doublet

MatchingDoublet[Sigma[β_1, α_1],Sigma[β_2,α_2],l_q,var,options]





Horizontal and vertical β-functions
first function (l_q=12 cm)
$\beta_h = \beta_v = 1.18026$ m
$\alpha_h = -\alpha_v = 2.39251$

Horizontal and vertical β-functions
second function (l_q=12 cm)
$\beta_h = 0.9952$ m, $\beta_v = 0.9939$ m
$\alpha_h = 2.0018$, $\alpha_v = -2.0005$

Example with Isochronous Period

Two step scenario:

1) Compute a channel providing the conditions of zero orbit dilation and vertical phase advance μ_v

2) Find periodic conditions for betatron functions

$$f_1 = \frac{-l\left(\cos\mu_h - \cos\mu_v + \sqrt{16[2-\cos\mu_h-\cos\mu_v]+[\cos\mu_h-\cos\mu_v]^2}\right)}{8[-2+\cos\mu_h+\cos\mu_v]}$$

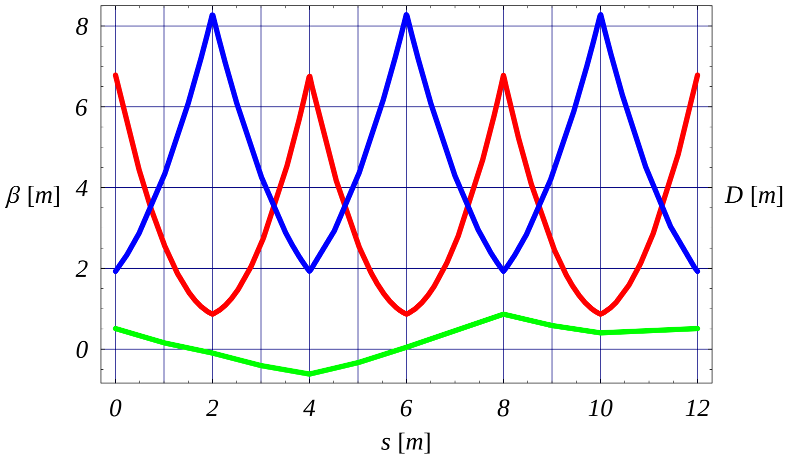
$$f_2 = \frac{-l\left(-\cos\mu_h + \cos\mu_v + \sqrt{16[2-\cos\mu_h-\cos\mu_v]+[\cos\mu_h-\cos\mu_v]^2}\right)}{8[-2+\cos\mu_h+\cos\mu_v]}$$

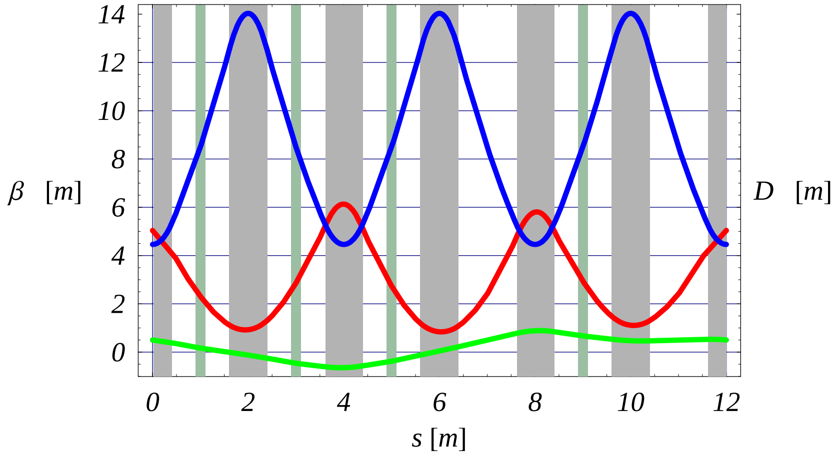
$$k_1 = \frac{1}{f_1 * l_q} \quad , \quad k_2 = \frac{1}{f_2 * l_q}$$

$\Rightarrow$ Minimization with the free parameters k_1, k_2

Example with Isochronous Period

IsoPeriod[n, l_q, l_b, MuV, Deflection, CellLength, MissingMagnet]





Horizontal and vertical β-functions and dispersion in thin lens model for n=3

Horizontal and vertical β-functions and dispersion with finite length elements (l_q=80 cm, l_b=20 cm)

Optimum Optical Systems

Following the same procedure, automatic elaboration of 15 new modules has been added (Doublet, Triplet, FODO, Wiggler, Inversor, Collins Insertions, etc.)

The code is OPEN, new contributions are welcome!

Goal $\Longrightarrow$ Encyclopaedia of modules, new functions are needed (wigglers, solenoids, etc.)

All functions are independent

◀VALUABLE TOOL FOR ACCELERATOR DESIGN▶

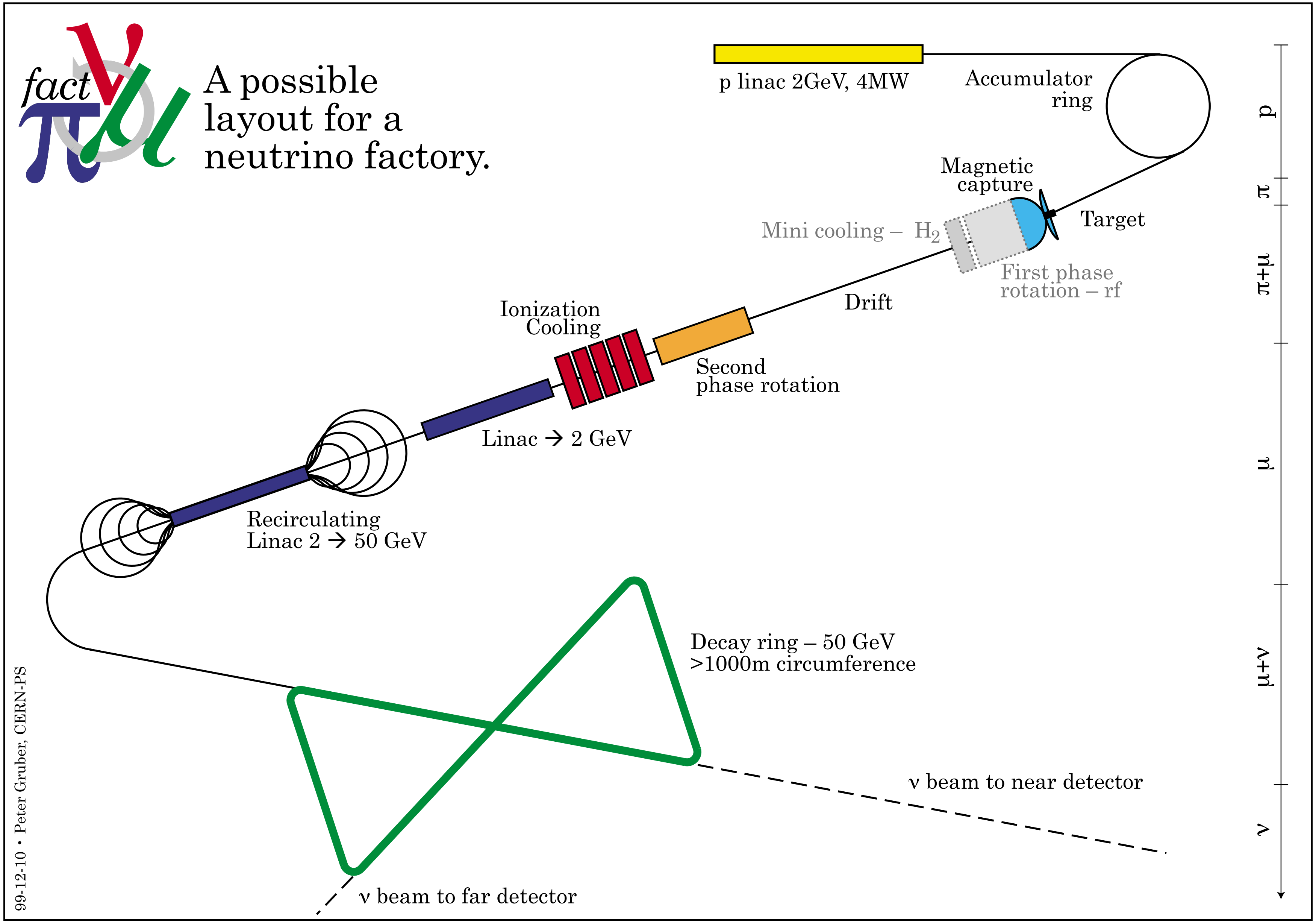
```
FODO[ff_, opt___Rule] :=
 Module[{ah, av, at, bg, beta, ch1, ch2, ch3, ch4, cobh, cobv, cx, f, le, n, p1, p2, pl, sih, siv},
  pl = If[SymbolicQ@N[{ff}], Identity, DisplayFunction /. {opt} /. Options[Plot]];
   {cobh, cobv, at, bg} = {BetaH, BetaV, PlotStyle, Background} /. {opt} /. Options[OpticsColors];
  {n, le} = {CellOrigin, CellLength} /. {opt} /. Options[OpticsModules];

  If[Abs@N[f] < .25, Return[Message[FODO::foc]]]; f = ff / le;
  ah = -4 f / Sqrt[16 f^2 - 1]; av = -ah; beta = av * le * (32 f^2 - 1) / (16 * f); cx = ah / (4 ff * Pi);
  ch1 = Channel[SS[le / 4], Q[2 ff], Q[2 ff], SS[le / 2], Q[-2 ff], Q[-2 ff], SS[le / 4]]; ch2 = ToVertical[ch1];

  {sih, siv} = {SigmaAll[ch1, Sigma[beta, ah]][[n]], SigmaAll[ch2, Sigma[beta, av]][[n]]};

   ch3 = RotateLeft[ch1, n - 1]; ch4 = ToVertical[ch3];
  If[pl === $DisplayFunction, p1 = BetaPlot[ch3, sih, PlotStyle -> {at, cobh}, DisplayFunction -> Identity];
   p2 = BetaPlot[ch4, siv, PlotStyle -> {at, cobv}, DisplayFunction -> Identity];

    {OpticsPlot -> Show[p1, p2, FilterOptions[Graphics, opt], Background -> bg, Frame -> True, RotateLabel -> False,
         FrameLabel -> {s  [m], β  [m],    , D  [m]}, PlotRange -> All, DisplayFunction -> $DisplayFunction], BeamLine -> ch3,
    SigmaEnd -> {sih, siv}, Chromaticity -> cx}, {BeamLine -> ch3, SigmaEnd -> {sih, siv}, Chromaticity -> cx}]]
```

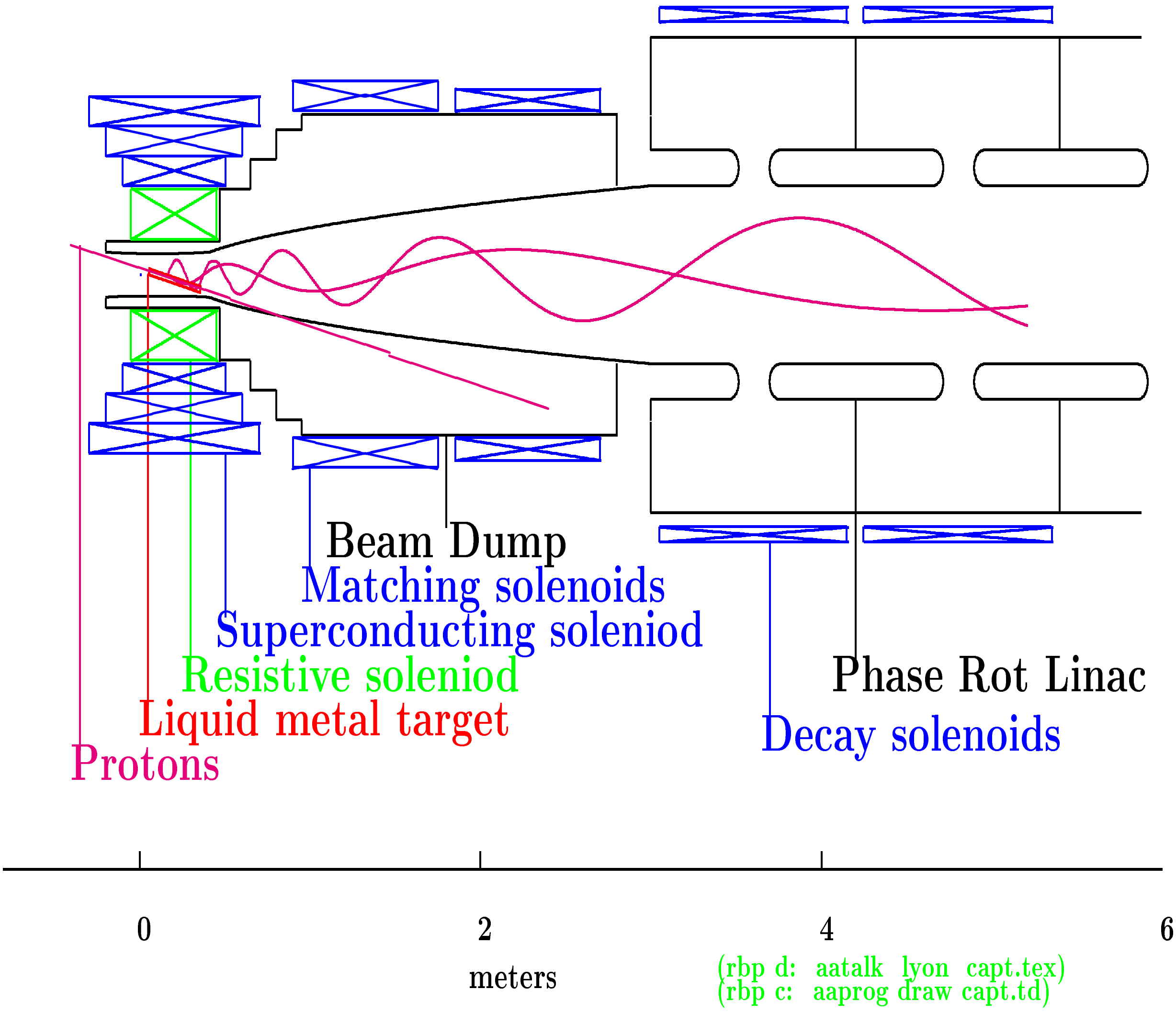
Neutrino Factory Design

General Goal $\Longrightarrow$ Produce a collimated, high flux neutrino beam from the decay of a muon beam circulating in a storage ring

Physics Goals $\Longrightarrow$ Study neutrino oscillations in distant detectors

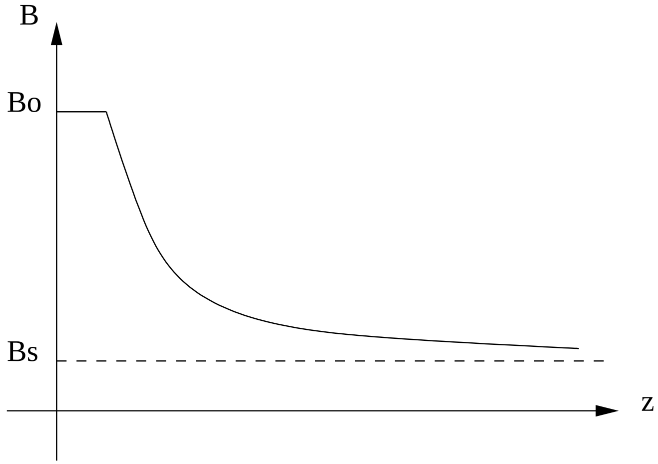
From the accelerator point of view, one generic scheme...(?)


fact
A possible layout for a neutrino factory.
p linac 2GeV, 4MW
Accumulator ring
Magnetic capture
Target
Mini cooling – H_2
First phase rotation – rf
Drift
Second phase rotation
Ionization Cooling
Linac → 2 GeV
Recirculating Linac 2 → 50 GeV
Decay ring – 50 GeV
>1000m circumference
ν beam to near detector
ν beam to far detector
p
π
π+μ
μ
μ+ν
ν
99-12-10 • Peter Gruber, CERN-PS



- **TARGET**: Liquid Metal Jet
- **CAPTURE**: 20 T Solenoid
- **DUMP**
- **MATCHING**
- **DECAY & PHASE ROT.**: 1.25 T

Capture Solenoid



Adiabatic Device

Field Law: $\frac{B_0}{1+\alpha z}$ $\alpha = \frac{\epsilon B_0}{P_0}$

Two Invariants: $\frac{P_\perp^2}{eB}$ and Br^2

$$B_0 = 20 \text{ T} \qquad B_s = 1 \text{ T}$$

$$\begin{cases} r_0 = \sqrt{\frac{B_s}{B_0}}a = 3 \text{ cm} \\ P_{\perp 0} = \frac{e\sqrt{B_s B_0}a}{2} = 100 \text{ MeV/c} \end{cases}$$

$$\begin{cases} r_{final} = a = 15 \text{ cm} \\ P_{\perp final} = \frac{eB_s a}{2} = 20 \text{ MeV/c} \end{cases}$$

Design Issues

Solenoid Issues:

- Transverse size $\simeq$ 1 m diameter
- Behaviour of RF cavities (for bunch rotation) inside magnetic field
- Cost

¿Is a standard quadrupole channel an alternative?

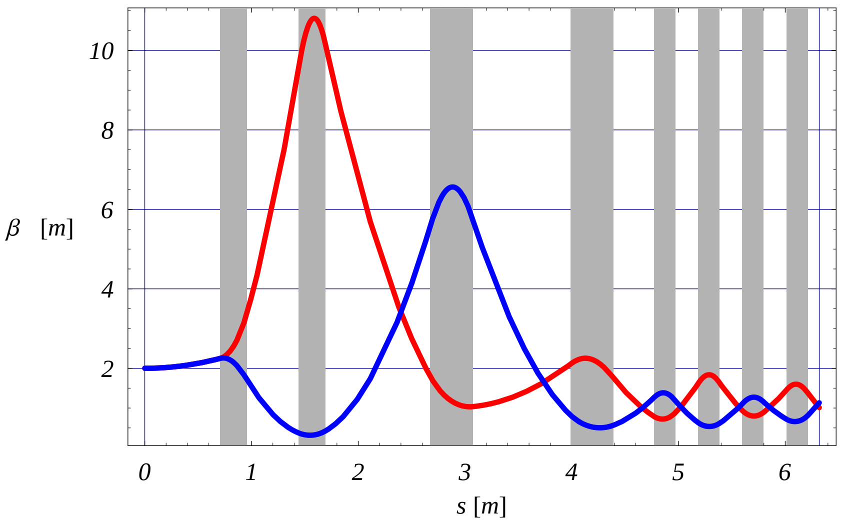
Quadrupole Issues:

- Match a symmetrical beam to an alternating gradient configuration
- Large beam radius because of large beam emittance ($\epsilon = rP_\perp/P$)
- Energy Spread (longitudinal momentum from 300 MeV/c to 500 MeV/c)

$\looparrowright$ **Two Solutions**

Transition Optics (1)

Beam Transport



→ Input: $\beta = 2$ m $\epsilon = 1.10^{-2}$ m.rad

→ Doublet from waist to crossover

Second quadrupole: B=1.8 T, L=25 cm

→ Doublet to match FODO cells at $\beta = 1$ m

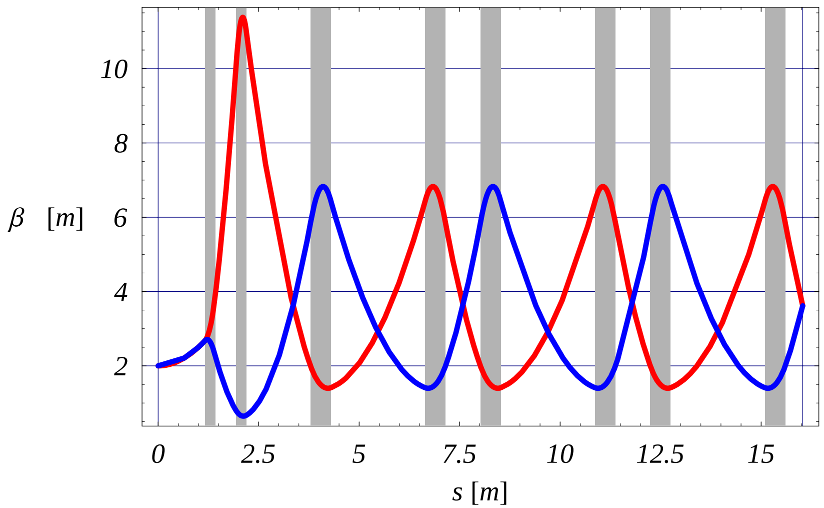
→ FODO cells with B=1.5 T

for L=20 cm, r=12 cm

Transition Optics (2)

With acceleration or bunch rotation

Collins Insertions: $L_{max} = \beta + \frac{1}{\gamma}$, $f = \frac{\alpha}{\gamma}$, $d = \frac{2l\alpha^2}{1+(\gamma l)^2}$ (quadrupole spacing)



→ Input: $\beta = 2$ m $\epsilon = 1.10^{-2}$ m.rad

→ Doublet from waist to crossover and up to $\beta = 3.6$ m

→ Collins Insertions to give room for RF 2.5 m with 1 m between quadrupoles B=0.4 T for L=50 cm, r= 25 cm (max)

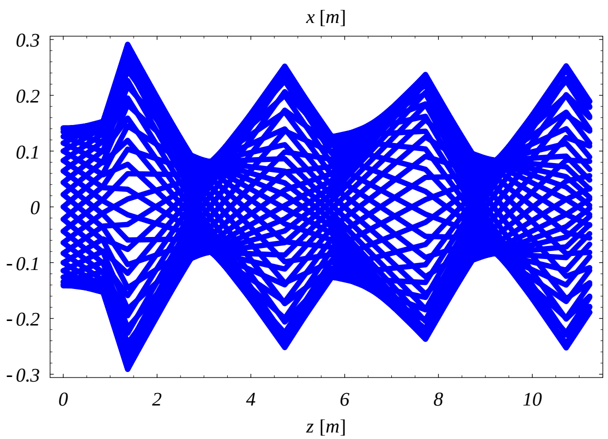
Conclusion: $P_{\perp} < 100$ MeV/c

Example of development activities (1)

Treatment of large chromatic and geometric aberrations

$$\epsilon = 10^{-2} \text{ m.rad !!!}$$
$$\text{p} \in [300\text{-}500 \text{ MeV/c}] \text{ !!!}$$

For off-momentum particles, reference orbit is not in general on the magnetic axis of the quadrupoles⇒ Non-linear tracking ($\sin\phi \neq \phi$, $\tan\phi \neq \phi$)



TTrack[x,θ] transverse vector in real space position-angle

LTrack[t,e] longitudinal vector in phase space time-energy

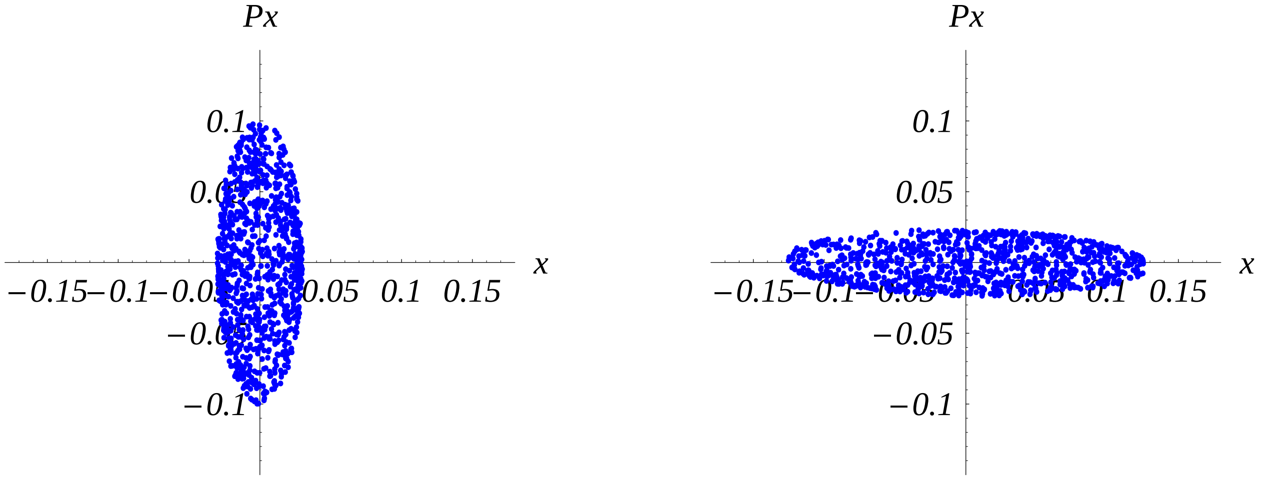
TrackAt[Channel,TTrack[x,θ],LTrack[t,e],LTrack[t_0, e_0]]

Example of development activities (2)

Tracking by integration of Lagrangian equations in Larmor frame for canonical variables in the adiabatic device→ AD[$B_0, B, length, radius, Momentum$]

(R.Chehab "Positron Sources" LAL/RT 92-17, Ph. Royer "Solenoidal Optics" PS/HP Note 99-12)

$$\begin{pmatrix} \xi \\ p_\xi \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{B_0}{B}}\cos\phi & \frac{2}{e\sqrt{B_0 B}}\sin\phi \\ -\frac{e\sqrt{B_0 B}}{2}\sin\phi & \sqrt{\frac{B}{B_0}}\cos\phi \end{pmatrix} \begin{pmatrix} \xi_0 \\ p_{\xi 0} \end{pmatrix} \quad \text{where} \quad \phi = \int_0^z \frac{eB}{2P} dz$$



Concluding Remarks

A - Unambiguous determination of real optical systems

B - Very useful tool for new machine design, tested for regular structures and insertions of high intensity proton machines, muon colliders, neutrino factory,...

C - The concept of *BeamOptics* is also applied to on-line optics corrections (ABS, Automatic Beam Steering and Shaping) for the LHC injectors chain, CEBAF, RHIC,...